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Chiral limit of light hadron mass in quenched staggered QCD

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We discuss chiral limit of light hadron mass from our quenched staggered calculations with a high lattice cutoff of a^{-1}~3.7 GeV at β=6.5 and a large lattice volume of $48^3 \times 64$. We added six heavier quark mass values of $m_q a$=0.0075, 0.015, 0.02, 0.03, 0.04 and 0.05 to the previously existing 0.01, 0.005, 0.0025, and 0.00125. An interesting curvature is observed in the m_π^2/m_q to m_q plot near $m_q a$=0.01.

We have been reporting our quenched staggered light hadron mass calculations for the past few years [1]. Our inverse squared coupling is set at β=6.5 corresponding to a high cutoff of a^{-1}~3.7 GeV. The lattice volume of $48^3 \times 64$ covers about 2.6 fm across for each space dimension and hence is comfortable enough even for our lightest pion with m_π~0.06 a^{-1}=220 MeV. We calculated staggered quark propagators using “corner” and “even” wall sources of a few different wall sizes and point sink, with quark mass values set at $m_q a$=0.01, 0.005, 0.0025, and 0.00125 for each of the 250 well-separated gauge configurations and then formed various light-hadron propagators. We did not see any significant autocorrelation among the hadron propagators, and our Jack-knife and [other] error analysis were all consistent with each other. So we could solidly draw various quantitative conclusions, of which most important are

1. flavor symmetry breaking among different staggered definitions of pion and ρ meson are smaller than the statistical errors,
2. m_π/m_ρ is as small as 0.27±0.01,
3. m_N/m_ρ is as small as 1.25±0.04.

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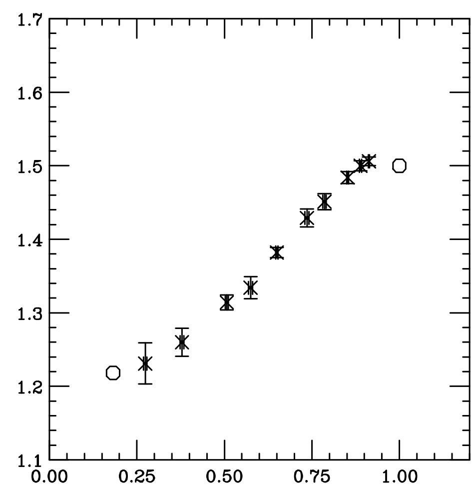

Figure 1. Edinburgh plot comparing the ratios m_N/m_ρ (vertical) and m_π/m_ρ (horizontal).

In addition we saw a possible sign of anomalous quenched chiral logarithm: the ratio m_π^2/m_q seemed to increase toward lighter quark mass values. Unlike with Wilson-fermion quarks where no good way to accurately determine the critical value of the hopping parameter is known, with staggered-fermion quarks we have a good control of chiral symmetry and hence of quark mass. So our chance in either establishing or excluding the presence of this anomalous effect is better. It is

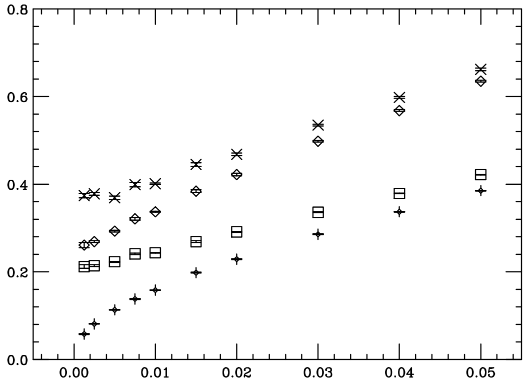

Figure 2. Δ ($\times$), N ($\diamond$), ρ ($\square$) and π (+) mass (vertical) vs bare quark mass (horizontal).

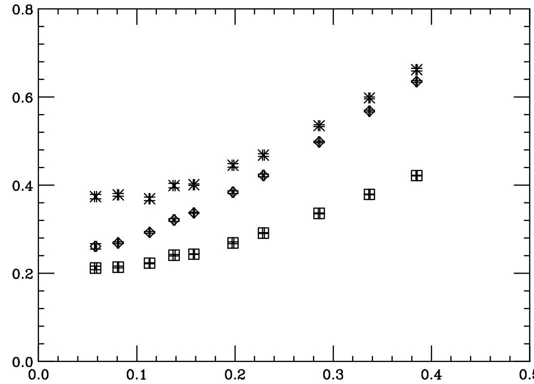

Figure 3. Δ ($\times$), N ($\diamond$) and ρ ($\square$) mass (vertical) vs π mass (horizontal)

this issue that we want to address in this poster.

Since it is prohibitively costly to push down the lightest quark mass value further, we decided to add several heavier quark mass values of $m_q a$=0.0075, 0.015, 0.02, 0.03, 0.04 and 0.05. So far we have accumulated propagator calculations for 30 gauge configurations evenly distributed over the 250 available ones. We summarize the obtained light hadron mass spectrum in Table 1 and in Figure 1 and 2.

The curvature in nucleon mass, as has been discussed since Lattice 97 [2] and is observed also in Figure 2, is probably not relevant for the anomalous quenched chiral log discussion because of renormalization in quark mass. More relevant is to compare the obtained pion mass with the other hadron mass estimates. In full QCD the correction for finite pion mass m_π should start with $O(m_\pi^2)$, but in quenched QCD it may start with $O(m_\pi)$ arising from the anomalous chiral log term. Actual behavior obtained from our lattice as shown in Figure 3 cannot yet distinguish these two cases: If we disregard chiral perturbation argument for the nucleon mass and try fitting to a naive form of $m_N = C_0 + C_1 m_q + C_2 m_q^2$, where m_q is the bare quark mass, we get C_0=0.248(2), C_1=9.3(2) and C_2=−31(4) with a confidence level of 91%. Similar naive fitting gives us C_0=0.354(2), C_1=5.4(2) and C_2=16(5) for the Δ resonance mass with confidence level of 8.3×10^{-10}, and C_0=0.200(1), C_1=4.6(1) and C_2=−3(2) with a confidence level of 9.5×10^{-4} for the ρ meson mass. On the other hand a fitting form of $m_N = C_0 + C_1 m_\pi + C_2 m_\pi^2$ gives C_0=0.204(3), C_1=0.662(4) and C_2=1.21(8) with a confidence level of 1.1×10^{-4} for the nucleon mass. This form gives equally bad fitting as the naive form for Δ resonance and the ρ meson.

On the other hand Gell'Mann-Oakes-Renner mass ratio m_π^2/m_q seems more suggestive. In full QCD we expect this ratio to behave like $\mu_0 + \mu_1 m_q a$ near the chiral limit $m_q a \to 0$ [3]. With the anomalous quenched chiral log present it would be modified by an additional $\mu' \ln m_q a$ behavior. Our current result, shown in Figure 4, seems to suggest there is this anomalous logarithmic contribution, though the statistics is not good enough yet. Finite-volume effect in m_π^2 may show up as a finite but non-zero intercept for m_π^2-m_q curve [4]. Fitting to the form of $m_\pi^2 = C_0 + C_1 m_q + C_2 m_q^2$ gives C_0=0.00066(96), C_1=2.3(1) and C_2=12(2) with a confidence level of 99%. Note that since our bare quark mass m_q in the current fitting range is small, higher order terms in m_q is irrelevant. The small (consistent with zero within error) intercept C_0 may suggest the absence of finite-volume effect. On the other hand, fitting to the form of $\ln m_\pi^2 =$

Table 1
Hadron mass estimates. Since heavier quarks are less sensitive to gauge field fluctuations, statistics for the additional heavier quark mass values are already good enough except for the two lighter ones of $m_q a$=0.0075 and 0.015. We will add more samples for these two and other heavier quark mass values.

$m_q a$	π	ρ	N	Δ
0.00125	0.0580(8)	0.212(4)	0.261(6)	0.374(5)
0.0025	0.0811(6)	0.214(2)	0.269(3)	0.378(3)
0.005	0.1131(5)	0.223(1)	0.293(2)	0.369(4)
0.0075	0.138(1)	0.241(2)	0.321(3)	0.399(5)
0.01	0.1582(5)	0.2434(8)	0.337(1)	0.401(2)
0.015	0.198(1)	0.269(2)	0.384(3)	0.445(4)
0.02	0.229(1)	0.291(1)	0.422(3)	0.468(3)
0.03	0.2857(8)	0.336(1)	0.498(2)	0.535(2)
0.04	0.3369(7)	0.379(1)	0.568(2)	0.598(2)
0.05	0.3850(6)	0.4216(9)	0.635(2)	0.662(3)

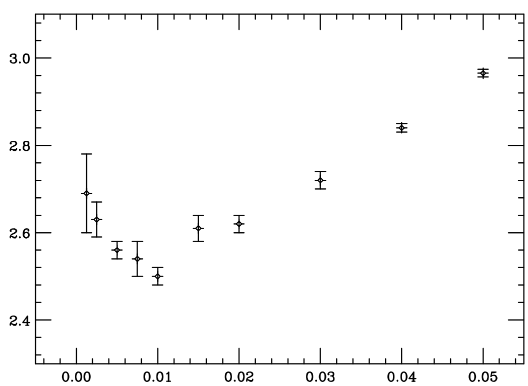

Figure 4. Gell'Mann-Oakes-Renner mass ratio m_π^2/m_q (vertical) vs bare quark mass m_q (horizontal).

$c+\ln m_q-\delta m_{\eta'}^2+dm_\pi^2+em_\pi^4$ [5] gives c=0.68(7), δ=0.005(2), d=2.8(6), e=−5(2) with a confidence level of 16%.

Conclusions: Even with the current small statistics for the added heavier quark mass values our investigation of the quenched chiral logarithm is already showing an interesting sign. It may become conclusive when the statistics is improved from the current 30 configurations to the target 250 ones, especially at the quark mass values of m_q = 0.0075 and 0.015 where the Gell'Mann-Oakes-Renner mass ratio m_π^2/m_q is showing interesting curvature.

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